

DESTABILIZING DIVERGENCES IN SUPERGRAVITY THEORIES

JONATHAN A. BAGGER

*Department of Physics and Astronomy
Johns Hopkins University
Baltimore, MD 21218*

Abstract

We study the stability of the gauge hierarchy in hidden-sector supergravity theories. We show that a destabilizing tadpole can appear if a theory has a gauge- and global-symmetry singlet with renormalizable couplings to the visible fields. We also find a quadratically divergent two-loop contribution to the supersymmetric effective potential. This term illustrates the difficulty of using the “LHC mechanism” to control quadratic divergences in theories with Planck-scale vevs.

1. Introduction

In a renormalizable theory, supersymmetry stabilizes the gauge hierarchy,

$$M_W \ll M_P, \quad (1)$$

by canceling all quadratic divergences. This cancellation persists even if supersymmetry is explicitly broken by certain soft operators [1]. These two facts have sparked an explosion of interest in the phenomenological application of supersymmetric gauge theories.

The problem is that any realistic theory is likely to be a *nonrenormalizable* effective theory, valid below some scale Λ . This might be the Planck scale, M_P , the string scale, M_X , or the unification scale, M_G [2]. For the purposes of this talk, we will take $\Lambda \simeq M_P$.

In a supersymmetric effective theory, the Kähler potential, K , and the superpotential, P , typically contain an infinite tower of nonrenormalizable terms, suppressed by the scale Λ . For the case at hand, this means

$$\begin{aligned} K &= \Phi^\dagger \Phi + \Phi^\dagger \Phi \left(\frac{\Phi + \Phi^\dagger}{M_P} \right) + \dots \\ P &= \Phi^3 + \frac{1}{M_P} \Phi^4 + \dots \end{aligned} \quad (2)$$

As we shall see, these nonrenormalizable terms can reintroduce quadratic divergences. These divergences have the potential to destabilize the gauge hierarchy [2] – [4].

In this talk I will report on work with Erich Poppitz and Lisa Randall [5] in which we clarify the conditions under which radiative corrections destabilize the gauge hierarchy. We will find a two-loop quadratically divergent contribution to the superspace effective potential. We shall see how this term can destabilize the hierarchy in supersymmetric theories with gauge- and global-symmetry singlets. We will also discuss how it affects the hierarchy in theories with Planck-scale vevs.

2. Destabilizing Divergences

Throughout this talk we will use a toy model to represent the minimal supersymmetric standard model. The toy model embodies all the essential physics that we wish to discuss. Therefore we shall restrict our attention to a single “Higgs” superfield, H , and take the superpotential, P , to be

$$P = \frac{1}{2} \mu H^2 . \quad (3)$$

With this potential, the Higgs has a mass $M_H \simeq \mu \simeq M_W$. (A discrete Z_2 symmetry replaces the gauge symmetry of the standard model. We assume that Z_2 is not broken for scales larger than M_W .)

The hierarchy is destabilized if radiative corrections lift $M_H \gg M_W$. In this model, simple superspace power counting indicates that the hierarchy is stable [3]. With more fields, however, the hierarchy can be destroyed. The potentially dangerous operators involve gauge- and global-symmetry singlet superfields.

Without loss of generality, we can distinguish two cases:

- Let N be a gauge- and global-singlet superfield, which couples directly to the Higgs,

$$P = \frac{1}{2} \mu H^2 + \frac{1}{2} m N^2 + \lambda N H^2 + \dots \quad (4)$$

Because of its renormalizable coupling to H , the field N is said to be in the visible sector. (For the purposes of this talk, we take $m = \mu$.)

- Let C be a gauge- and global-singlet hidden-sector superfield (*un champ du secteur caché*), which couples to the visible sector through terms suppressed by the Planck mass, M_P ,

$$P = \frac{1}{2} \mu H^2 \left[1 + \left(\frac{C}{M_P} \right)^n + \dots \right] . \quad (5)$$

Typically, the visible-sector fields H and N are assigned weak-scale vevs,

$$\begin{aligned} \langle H \rangle &\lesssim M_W + \theta \theta M_W^2 \\ \langle N \rangle &\lesssim M_W + \theta \theta M_W^2 , \end{aligned} \quad (6)$$

while the field C can have a larger vev,

$$\langle C \rangle \lesssim M_P + \theta \theta M_S^2 , \quad (7)$$

where $M_S^2 \simeq M_W M_P$ denotes the scale of supersymmetry breaking. The vev $\langle C \rangle$ can be fixed by the hidden-sector potential, or it can be free, as with a string modulus, in which case we denote C by T .

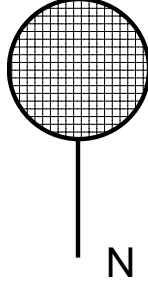


Fig. 1. A visible-sector tadpole diagram.

The vevs (6) and (7) are typical tree-level vevs for the fields N and C . They preserve the hierarchy, as can be seen by substituting into (4) and (5). The lowest components of the superfield vevs give a supersymmetric renormalization of μ , while the highest components induce a supersymmetry-violating mass for the scalar component, h , of the Higgs superfield, H .

From these expressions we see that the hierarchy is destabilized if loop corrections induce vevs of the order

$$\langle N \rangle \simeq M_S + \theta \theta M_S^2 \quad (8)$$

or

$$\langle C \rangle \simeq M_P + \theta \theta M_P^2 . \quad (9)$$

These vevs are dangerous and must be avoided if supersymmetry is to solve the gauge hierarchy problem.

3. Destabilizing Divergences at One Loop

3.1 General Formalism

In flat space, the supersymmetric action is given by

$$S = \int d^4\theta K + \left[\int d^2\theta P + h.c. \right] , \quad (10)$$

where K is the Kähler potential, and P is the superpotential of the supersymmetric theory.

In curved space, the situation is more complicated. The superspace kinetic term is given by

$$- 3M_P^2 \int d^4\theta E e^{-K/3M_P^2} , \quad (11)$$

while the superspace superpotential is

$$\int d^2\theta \mathcal{E} P + h.c. \quad (12)$$

In these expressions, E and \mathcal{E} are superdeterminants of the supervielbein, superspace generalizations of the density $\det e_m^a$ familiar from general relativity.

The superspace action (11) plus (12) is manifestly supersymmetric. It is also invariant under super-Kähler-Weyl transformations, because

$$E \sim \Sigma^+ \Sigma , \quad \mathcal{E} \sim \Sigma^3 , \quad (13)$$

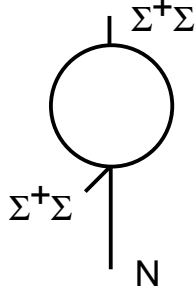


Fig. 2. In the minimal case, the one-loop quadratically divergent contributions cancel between the two $\Sigma^+\Sigma$ contributions.

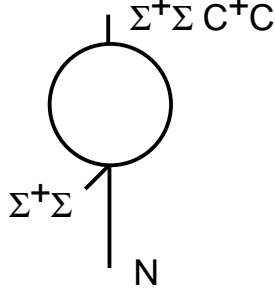


Fig. 3. In the nonminimal case, the one-loop quadratically divergent contributions do not cancel.

where Σ is the superconformal compensator. When supersymmetry is broken, Σ plays the role of a spurion,

$$\langle \Sigma \rangle \simeq 1 + \theta\theta \frac{M_S^2}{M_P}. \quad (14)$$

For hidden-sector scenarios, with $M_S^2 \simeq M_W M_P$, $\langle \Sigma \rangle$ contributes to the soft masses of the visible-sector particles. (When $M_S \simeq M_W$, the vev of Σ can be ignored.)

3.2 Visible-Sector Tadpoles

We are now ready to begin our analysis of destabilizing divergences in supergravity theories. We first consider the case of a gauge- and global-symmetry singlet in the visible sector. By power counting, it is not hard to see that the dangerous diagrams are tadpoles (Fig. 1), which scale like

$$\begin{aligned} \delta S &\simeq \Lambda \int d^4\theta \, E N \\ &\simeq \Lambda \int d^4\theta \, \Sigma^+ \Sigma N \\ &\simeq M_S^2 \int d^2\theta \, \Sigma N. \end{aligned} \quad (15)$$

In hidden-sector models, such a tadpole will induce a vev of order M_S^2 for the highest component of N . This vev is dangerous because it gives a mass of order M_S to the Higgs scalar, h .

To determine whether such a tadpole arises, let us first consider the minimal case, in which there is no coupling between the visible and invisible sectors. Therefore we take the Kähler potential to be

$$K = N^+ N + C^+ C + H^+ H \left(1 + \frac{N + N^+}{M_P} \right) + \dots \quad (16)$$

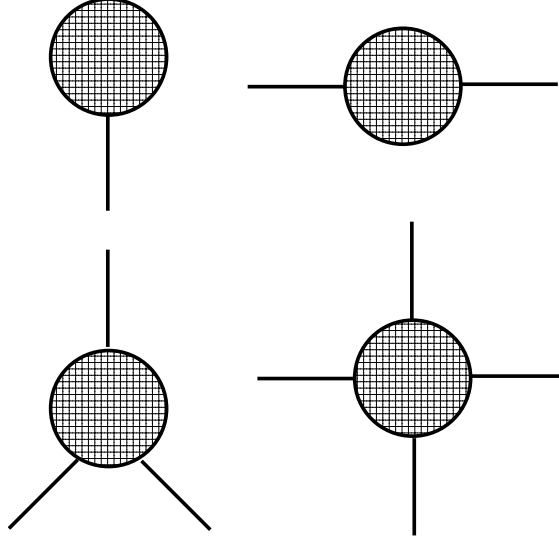


Fig. 4. The superfield effective potential involves vevs of the hidden-sector fields.

At one loop, there is one potentially dangerous superspace diagram, as shown in Fig. 2. Each $\Sigma^+\Sigma$ insertion induces a quadratic divergence. However, the two divergences exactly cancel, so the hierarchy is stable [4]. It is necessary to go to two loops to see whether the cancellation is natural, or whether it is an accident of the one-loop approximation.

Let us now consider the nonminimal case, in which there are couplings between the visible and hidden-sector fields. We take the Kähler potential to be given by

$$K = N^+N + C^+C + H^+H \left(1 + \frac{N + N^+}{M_P} + \frac{CC^+}{M_P^2} \right) + \dots \quad (17)$$

As before, there is one potentially dangerous diagram, as shown in Fig. 3. Now, however, when C gets a vev, the extra term in K spoils the cancellation between the two quadratic divergences. A one-loop tadpole is induced [3, 4]

$$\begin{aligned} \delta S &\simeq \frac{\Lambda^2}{M_P} \int d^4\theta \, E N \\ &\simeq M_P \int d^4\theta \, E N , \end{aligned} \quad (18)$$

and the hierarchy is destabilized. Visible-sector singlets can destabilize the hierarchy if the visible and invisible sectors couple, even by nonrenormalizable terms suppressed by M_P !

3.3 Effective Potential

Of course, models free from visible-sector singlets are not necessarily free from destabilizing divergences. One must still consider the hidden-sector singlets C . If $\langle C \rangle \lesssim M_S + \theta\theta M_S^2$, as in dynamical hidden-sector models, the hierarchy is safe. But if C is a modulus T , with a Planck-scale vev, the usual power counting rules break down, and all graphs are potentially dangerous (Fig. 4). In this case one must compute the full effective potential and make sure that the field-dependent quadratic divergences vanish.

The one-loop correction to the supersymmetric effective potential has been known for some time [6]. It is given by

$$\begin{aligned} \delta S &\simeq \Lambda^2 \int d^4\theta \, E \log \det K^I{}_J + \dots \\ &\simeq \Lambda^2 R^I{}_J P_I \bar{P}^J + \dots , \end{aligned} \quad (19)$$

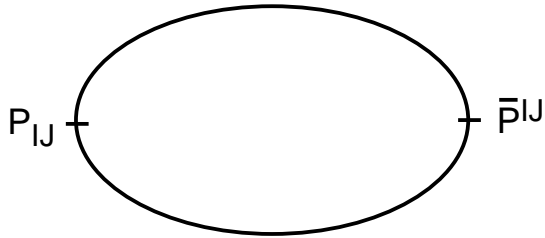


Fig. 5. A one-loop contribution to the superfield effective action.

where P_I is the Kähler-covariant derivative of the superpotential, and R^I_J is the Ricci tensor of the Kähler manifold specified by K . If R^I_J does not vanish, the vacuum can be destabilized,

$$\begin{aligned} \langle T \rangle &\rightarrow 0 \\ \langle T \rangle &\rightarrow M_P + \theta \theta M_P^2. \end{aligned} \quad (20)$$

In this case the gravitino mass is driven to zero or M_P . Therefore if $R^I_J \neq 0$, radiative corrections can lead to a radically new vacuum.

In a recent paper, Ferrara, Kounnas and Zwirner [7] imposed a geometrical condition on K which ensures that $R^I_J = 0$. The resulting models – which they called LHC models – are automatically free from one-loop quadratic divergences. One would like to know whether the one-loop cancellation persists to higher loops, and if not, to find the conditions that stabilize the hierarchy.

4. Destabilizing Divergences at Two Loops

4.1 General Formalism

These questions provide the motivation for computing the two-loop supersymmetric effective potential. The full calculation is rather involved, so we will focus on one important piece. We will show that there is a two-loop, superpotential-dependent, quadratically divergent contribution to V_{eff} . This term

- destabilizes the hierarchy in models with visible-sector singlets, and
- illustrates the difficulty of maintaining hierarchy in models with moduli.

To check our techniques, and to get warmed up, we first compute the one-loop logarithmically divergent diagram shown in Fig. 5. We find

$$\delta S \simeq \frac{\log \Lambda}{16\pi^2} \int d^4\theta \, e^{2K/3M_P^2} \, \Sigma^+ \Sigma \, P_{IJ} \, \bar{P}^{IJ}. \quad (21)$$

This term is super-Kähler-Weyl invariant, as required. When appropriately covariantized, it is also locally supersymmetric. In components, it agrees with the result of Gaillard and Jain [8]. And when $\Sigma^+ \Sigma$ gets a vev, it gives the correct β -functions for the soft supersymmetry-breaking masses.

With this experience, we are ready to compute the two-loop diagram shown in Fig. 6. Following the steps outlined in Ref. [5], we find (See also [9].)

$$\delta S \simeq \frac{1}{6} \frac{\Lambda^2}{(16\pi^2)^2} \int d^4\theta \, e^{K/M_P^2} \, P_{IJK} \, \bar{P}^{IJK}, \quad (22)$$

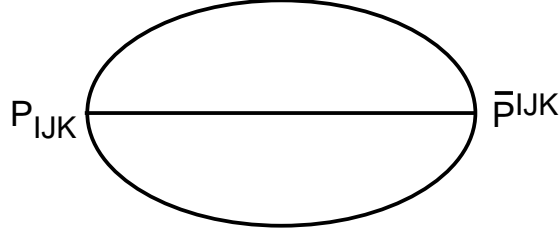


Fig. 6. A two-loop contribution to the superfield effective action.

where Λ is a momentum-space cutoff. As before, this expression is super-Kähler-Weyl invariant. It can be made locally supersymmetric with the help of the supergravity multiplet.

4.2 Visible-Sector Tadpoles

Let us use this result to revisit each of the dangerous cases discussed previously. For the case of the visible-sector singlet, we can use the field redefinition

$$H \rightarrow H \left(1 - \frac{N}{M_P}\right) \quad (23)$$

to write (4), (16) and (17) in the following form,

$$\begin{aligned} K &= N^+ N + C^+ C + H^+ H + \mathcal{O}(1/M_P^2) \\ P &= \frac{1}{2} \mu H^2 + \frac{1}{2} m N^2 + \lambda' N H^2 - \lambda'' \frac{N^2 H^2}{M_P} + \mathcal{O}(1/M_P^2). \end{aligned} \quad (24)$$

Substituting into (22), we find the following destabilizing divergence,

$$\begin{aligned} \delta S &\simeq \Lambda^2 \int d^4\theta e^{K/M_P^2} P_{IJK} \bar{P}^{IJK} \\ &\simeq \Lambda^2 \int d^4\theta e^{K/M_P^2} \frac{N}{M_P} + \dots \\ &\simeq \frac{1}{M_P} \int d^4\theta C^+ C N \\ &\simeq M_W M_S^2 n. \end{aligned} \quad (25)$$

This is a two-loop destabilizing divergence. It indicates that the one-loop cancellation was purely accidental, and that visible-sector singlets are always dangerous if they have renormalizable couplings to the other visible-sector fields.

4.3 Effective Potential

Finally, let us discuss two-loop effective potential. Equation (22) contains a two-loop field-dependent quadratic divergence – a divergence which depends on the parameters of the visible-sector superpotential. If the hierarchy is to be stable, the divergence must be canceled, either by contributions from the hidden sector or by higher string modes.

In either case, a string miracle is required. The hidden sector or the higher modes must know about visible-sector parameters like m_t , m_e , V_{cb} . These interrelations cannot be understood in terms of the low energy effective theory. Our result shows that string moduli with Planck-scale vevs have the potential to destabilize the hierarchy. Whether they do, or not,

depends on the miracles of string theory.

5. Conclusion

In this talk we considered questions of naturalness in supersymmetric effective theories. In particular, we studied the potentially destabilizing quadratic divergences that are induced at one- and two-loops in the effective potential.

We first discussed the possible generation of divergent tadpole diagrams. We explained why gauge- and global-symmetry singlets do *not* develop tadpoles at one-loop order, if the Kähler potential is minimal. We then showed that at two loops, quadratically divergent tadpoles can indeed appear.

Our results indicate that the one-loop cancellation is an accident of the one-loop approximation. This conclusion is in accord with our notions of naturalness because there is no symmetry that would forbid a divergent tadpole. Since there is no symmetry, it has to appear, and indeed it does.

We also discussed the LHC models of Ferrara, Kounnas and Zwirner. These models rely on a cancellation of the one-loop field-dependent quadratic divergences. Again, since this cancellation is not related to a symmetry of the theory, we expect a contribution to arise at higher loops. Indeed, at two loops we found that the cancellation is spoiled by terms that depend on the superpotential of the visible sector.

Clearly, if LHC models are to work, there must be some string-induced conspiracy which cancels the Yukawa-dependent divergence. Such a cancellation would be difficult to understand at the level of effective field theory.

Our conclusions can be readily generalized to higher loops. The superpotential-dependent divergences can be guessed by induction. At one loop, we found logarithmically divergent contributions to the component Kähler potential which go like

$$\log \Lambda \, e^{K/M_P^2} \left(P_{IJ} \bar{P}^{IJ} + \frac{1}{M_P^2} P_I \bar{P}^I + \frac{1}{M_P^4} P \bar{P} \right) . \quad (26)$$

At two loops, we found quadratically divergent terms such as

$$\Lambda^2 \, e^{K/M_P^2} \left(P_{IJK} \bar{P}^{IJK} + \frac{1}{M_P^2} P_{IJ} \bar{P}^{IJ} + \frac{1}{M_P^4} P_I \bar{P}^I + \frac{1}{M_P^6} P \bar{P} \right) . \quad (27)$$

Therefore at three loops, we expect quartically divergent terms of the form

$$\Lambda^4 \, e^{K/M_P^2} \left(P_{IJKL} \bar{P}^{IJKL} + \frac{1}{M_P^2} P_{IJK} \bar{P}^{IJK} + \frac{1}{M_P^4} P_{IJ} \bar{P}^{IJ} + \frac{1}{M_P^6} P_I \bar{P}^I + \frac{1}{M_P^8} P \bar{P} \right) . \quad (28)$$

In each case, the leading term comes from a rigid supersymmetry graph, while the other terms come from graphs with supergravity fields in the loops.

Taking $\Lambda \simeq M_P$, we see that the three-loop terms induce new possibilities for destabilizing divergences. For example, the $P_{IJKL} \bar{P}^{IJKL}$ term can also contain a quadratically divergent tadpole. This implies we must expect new superpotential-dependent divergences at each order of perturbation theory. The cancellation of quadratic divergences requires a grand conspiracy between terms at all orders in the loop expansion. Presumably this cancellation is related to the cosmological constant problem, about which we have nothing to say.

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